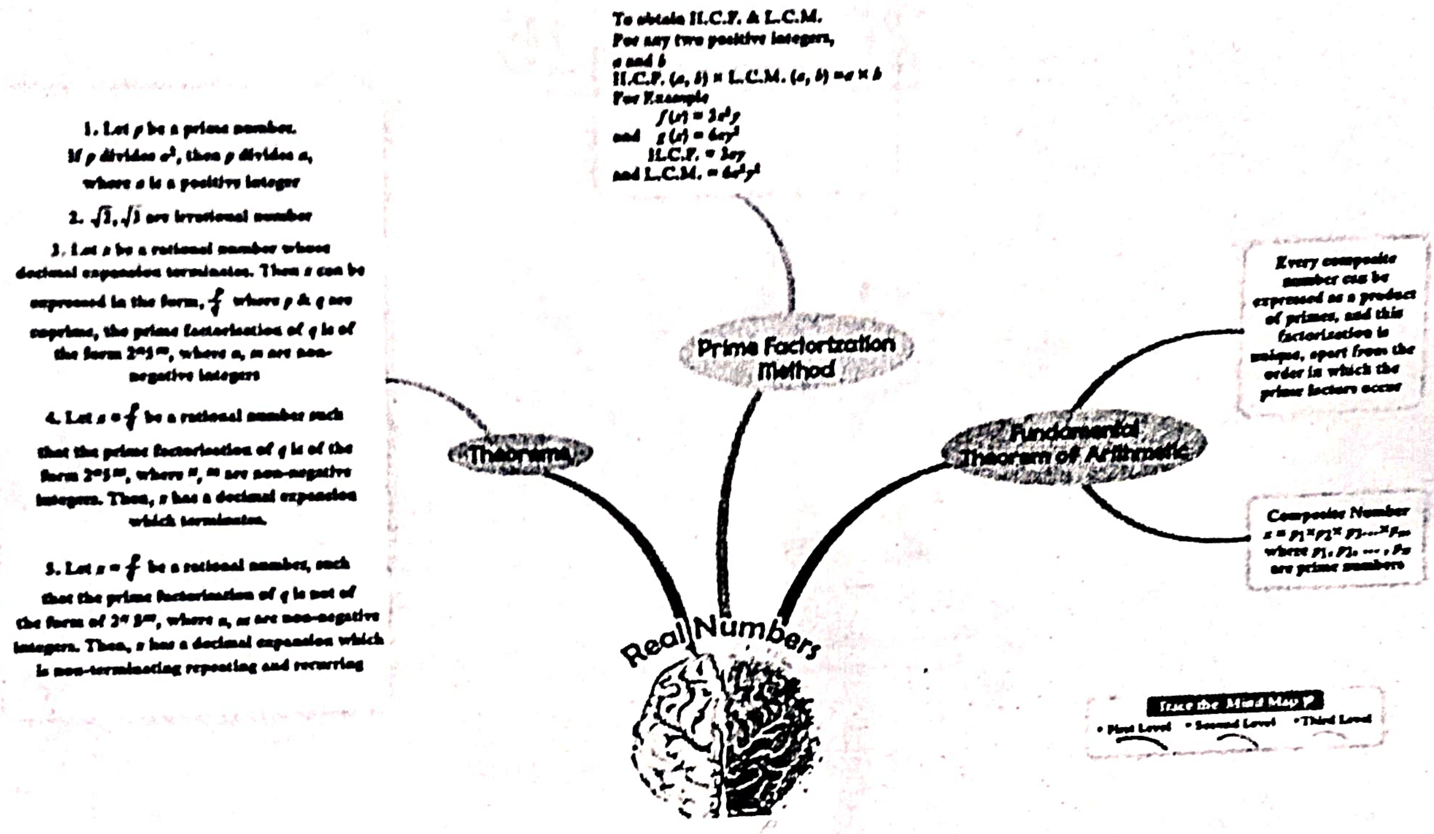
CHAPTER-1
REAL NUMBERS
MIND MAPPING



GIST OF THE CHAPTER

1. Real Numbers: Include both rational and irrational numbers.
2. Fundamental Theorem of Arithmetic: Every Composite number can be expressed as a product of primes, and the factorisation is unique except for the order of the prime factors.
3. Prime Factorization Applications: Useful for finding HCF and LCM.
4. Rational Numbers
5. Irrational numbers and it's properties.

DEFINITION

1. Real Numbers: Set of numbers that can be represented on number line. Include both rational numbers and irrational numbers.
2. Natural numbers (1, 2, 3,......), Whole numbers (0, 1, 2, 3,......), Integers (...., -2, -1, 0, 1, 2, ...)
3. Rational numbers: Number that can be expressed in the form of $\frac{p}{q}$ where $q \neq 0$
4. Irrational numbers: Numbers that cannot be expressed as $\frac{p}{q}$, like $\pi, \sqrt{2}$

FORMULA

HCF AND LCM FORMULA (For two numbers)

HCF × LCM = Product of two numbers

HCF (a, b) × LCM (a, b) = a × b

HCF AND LCM FORMULA (For three numbers)

HCF (a, b, c) = HCF (HCF (a, b), c)

LCM (a, b, c) = LCM (LCM (a, b), c)

Step 1: Find HCF /LCM of any two numbers (say, a and b)

Step 2: Find HCF/LCM of that result with the third number (c).

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1. The HCF and the LCM of 12, 21 ,15 respectively is

(a)3 ,140 (b) 12, 420 (c) 3, 420 (d) 420,3

Ans. (c) 3, 420

2. The correct prime factorisation of 98 is

(a) $2^2 \times 7$ (b) $2^3 \times 7$ (c) $2^2 \times 7^2$ (d) 2×7^2

Ans. (d) 2×7^2

3. The greatest possible speed at which a man can walk 135 km and 225 km in exact number of hours is:

(a) 5 km/hr (b) 15 km/hr (c) 65 km/hr (d) 45 km/hr

Ans. (d) 45 km/hr

4. The expression $1\times 2\times 3 \times 7\times 11 +1$ is a

(a) Prime no. (b) Composite no. (c) Square number (d) Neither prime nor composite

Ans. (b) Composite no.

5. Which of the following is an incorrect statement?

(a) π is an irrational (b) 2 is a rational

(c) Every rational number is a real number (d) Every real number is a rational number

Ans. (d) Every real number is a rational number

6. LCM of smallest prime number and smallest 2 digit number is

(a) 2 (b) 4 (c) 10 (d) 20

Ans. (c) 10

7. If $a = p^2q$ and $b = pq^2$ then LCM will be?

(a) p^2q^2 (b) pq (c) pq^2 (d) pq^3

Ans. (a) p^2q^2

8. Find the product of HCF and LCM of (32,28) using relationship between HCF and LCM of two numbers

(a) 256 (b) 840 (c) 832 (d) 896

Ans. (d) 896

9. If $a = x^3y^2z^2$, $b = x^2y^2z^3$, and $c = x^3y^2z^n$ and the LCM (a,b,c) = $x^3y^2z^5$ then the value of n is:

(a) 3 (b) 2 (c) 5 (d) 1

Ans. (c) 5

10. The largest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively is:

(a) 63 (b) 36 (c) 34 (d) 45

Ans. (a) 63

ASSERTION AND REASONING QUESTIONS

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true

1. **Assertion(A):** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162.

   Reason(R): If a and b are two positive integers, then H.C.F. × L.C.M. = a × b.

   Ans. (d)

2. **Assertion(A):** The HCF of two numbers is 2 and their LCM is 15

   Reason(R): HCF is a factor of LCM.

   Ans. (d)

3. **Assertion(A):** $\frac{13}{\ldots}$ is a terminating decimal fraction.

Reason(R): If $q = 2^n.5^m$ where n and m are non-negative integers, then p/q is a terminating decimal fraction.
Ans. (a)

4. **Assertion(A):** 2 is an example of a rational number.
Reason(R): The square roots of all positive integers are irrational numbers.
Ans. (c)
5. **Assertion (A):** If the HCF of two numbers is 5 and their product is 150, then their LCM is 40.
Reason(R): For any two positive integers p and q, HCF (p, q) × LCM (p, q) = p × q.
Ans. (d)
6. **Assertion (A):** Every positive even integer is a multiple of 2.
Reason (R): The Fundamental Theorem of Arithmetic ensures the unique prime factorization of integers greater than 1.
Ans. (b)
7. **Assertion (A):** The decimal expansion of $\frac{7}{8}$ is terminating.
Reason (R): A rational number has a terminating decimal expansion if its denominator has only powers of 2 and/or 5 in prime factorization.
Ans. (a)
8. **Assertion (A):** $\sqrt{5}$ is an irrational number.
Reason (R): The square root of any non-perfect square natural number is irrational.
Ans. (a)
9. **Assertion (A):** If the HCF of two numbers is 1, they are said to be co-prime.
Reason (R): The LCM of two co-prime numbers is always equal to their product.
Ans. (a)
10. **Assertion (A):** The decimal expansion of $\frac{1}{6}$ is non-terminating and repeating.
Reason (R): A rational number has a non-terminating repeating decimal if the denominator has prime factors other than 2 or 5.
Ans. (a)

VERY SHORT ANSWER TYPE QUESTIONS (2MARKS QUESTIONS)

1. Prove that $\sqrt{3}$ is an Irrational number.
Ans. Let us assume that $\sqrt{3}$ is a rational number, $\sqrt{3} = \frac{a}{b}$, where a and b are co-primes. Squaring both side $(\sqrt{3})^2 = a^2 / b^2$
$3b^2 = a^2$, (3 divides a^2, 3 divides a) Let us consider a = 3c, by putting the value of a we get $b^2 = 3c^2$. (3 divides b^2, 3 divides b). so 3 is a common factor of both a and b, which is a contradict our assumption, Hence, $\sqrt{3}$ is an Irrational
2. Find the HCF and LCM of smallest prime number and smallest composite number.
Ans. HCF=2, LCM=4
3. The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45 find the other number.
Ans. HCF×LCM= Product of two nos. so the other no. is 72.
4. If product of two coprime numbers is 217, then find their LCM.
Ans. LCM=217
5. Explain Why (17 × 11 × 2 + 17 × 11 × 5) is a composite number?
Ans. Given no. is a product of more than two prime nos.
6. Find the LCM and HCF of 8 and 64
Ans. LCM=64, HCF=8
7. Given that HCF (252, 594) = 18, find LCM (252, 594).
Ans. LCM= 8316
8. Find the prime factorization of 2120.
Ans. $2^3 \times 5 \times 53$.

9. Show that any number of the form 4^n can never end with the digit 0.
Ans: If 4^n ends with 0 then it must have 5 as a factor. But we know the only prime factor of 4^n is 2. Also, we know from the fundamental theorem of arithmetic that prime factorization of each number is unique. Hence 4^n can never end with the digit 0.
10. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, then find the numbers.
Ans. The first no. =HCF ×first ratio= 13×15= 195
The second no. =HCF × first ratio= 13×11= 143

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1. Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11, and 15 respectively.
Ans: 398 – 7 = 391, 436 – 11 = 425, 542 – 15 = 527,
HCF of 391, 425, 527 = 17
2. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?
Ans: Minimum distance= LCM of 40,42,45= 2520
3. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280 then find other number.
Ans: HCF× LCM=product of two nos., LCM=14×HCF
it is given that HCF+LCM=600, by using the above we get HCF=40 and the other no. is =80.
4. Find the least positive integer which is divisible by first 5 natural numbers.
Ans: The first five natural nos. are 1,2,3,4,5. The least positive integer divisible by the first 5 natural nos.is 60.
5. Show that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number, if $\sqrt{15}$ is irrational number?
Ans: By using the formula of $(a+b)^2$ we get $8+2\sqrt{15}$. We know that the sum of rational and irrational is always irrational.
6. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?
Ans: HCF (616, 32) will give the maximum number of columns in which they can march.
$616 = 2^3 \times 7^1 \times 11^1$, $32 = 2^5$ The HCF (616, 32) = $2^3 = 8$.
Therefore, they can march in 8 columns each.Type equation here.
7. Prove that $3+2\sqrt{5}$ is irrational.
Ans: Let us assume on the contrary that $3+2\sqrt{5}$ is rational.
Then, there exist co-prime positive integers a and b such that $3+2\sqrt{5}=\frac{a}{b}$
$\Rightarrow 2\sqrt{5} = a/b - 3 \Rightarrow \sqrt{5} = \frac{a-3b}{2b}$
Since, a and b are integers and thus $\frac{a-3b}{2b}$ is rational number. Thus $\sqrt{5}$ is rational
But this contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect.
Hence, $3+2\sqrt{5}$ is an irrational number.
8. Prove that $5-\sqrt{3}$ is an irrational number.
Ans: Let us assume on the contrary that $5-\sqrt{3}$ is rational.
Then, there exist co-prime positive integers a and b such that $5-\sqrt{3}=\frac{a}{b}$
$\Rightarrow 5-\frac{a}{b}=\sqrt{3} \Rightarrow \frac{5b-a}{b}=\sqrt{3}$.
Since, a and b are integers and thus $\frac{5b-a}{b}$ is rational number.
Thus $\sqrt{3}$ is rational but this contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.
Hence, $5-\sqrt{3}$ is an irrational number.
9. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after

what time will they next toll together?
Ans: $9 = 3^2$, $12 = 2^2 \times 3$, $15 = 3 \times 5$, LCM $= 2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 = 180$ minutes or 3 hours
They will next toll together after 3 hours.

10. Two tankers contain 850 liters and 680 liters of petrol. Find the maximum capacity of a container which can measure the petrol of each tanker in the exact number of times.
Ans: To find the maximum capacity of a container which can measure the petrol of each tanker in the exact number of times, we find the HCF of 850 and 680.
$850 = 2 \times 5^2 \times 17$, $680 = 2^3 \times 5 \times 17$
HCF $= 2 \times 5 \times 17 = 170$, Maximum capacity of the container $= 170$ liters.

LONG ANSWER TYPE QUESTIONS (5 MARK QUESTIONS)

1. The traffic light at three consecutive different road crossing change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7am, at what time will they change simultaneously again?
Ans: Here we have to find the LCM of 48, 72 and 108 first then, convert the LCM from seconds to minutes and seconds. Finally, add the result to 7:00 am. The traffic lights will change simultaneously again at 7:07: 12 am.

2. The length, breadth, and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.
Ans: To find the length of the longest rod that can measure the dimensions of the room exactly, we have to find HCF.
L, Length $= 8$ m 50 cm $= 850$ cm $= 2^1 \times 5^2 \times 17$
B, Breadth $= 6$ m 25 cm $= 625$ cm $= 5^4$
H, Height $= 4$ m 75 cm $= 475$ cm $= 5^2 \times 19$
HCF of L, B and H is $5^2 = 25$ cm
Length of the longest rod $= 25$ cm

3. In a school, there are two Sections A and B of class X. There are 48 students in Section A and 60 students in Section B. Determine the least number of books required for the library of the school so that the books can be distributed equally among all students of each Section.
Ans: Since the books are to be distributed equally among the students of Section A and Section B. therefore, the number of books must be a multiple of 48 as well as 60.
Hence, required no. of books is the LCM of 48 and 60.
$48 = 2^4 \times 3$, $60 = 2^2 \times 3 \times 5$
LCM $= 2^4 \times 3 \times 5 = 16 \times 15 = 240$,
Hence, required number of books is 240.

4. There are 104 students in class X and 96 students in class IX in a school. In a house examination, the students are to be evenly seated in parallel rows such that no two adjacent rows are of the same class.
(a) Find the maximum number of parallel rows of each class for the seating arrangement.
(b) Also, find the number of students of class IX and also of class X in a row.
(c) What is the objective of the school administration behind such an arrangement?
Ans: The HCF of 104 and 96 is 8. (a) The maximum number of parallel rows of each class is 8
(b) The number of students of class IX in a row is 12, and the number of students of class X in a row is 13.
(c) The objective of school administration behind such an arrangement is fair and clean examination, so that no student can take help from any other student of his/her class.

5. Dudhnath has two vessels containing 720 ml and 405 ml of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled.
Ans: 1st vessel $= 720$ ml; 2nd vessel $= 405$ ml
We find the HCF of 720 and 405 to find the maximum quantity of milk to be filled in one glass.
$405 = 3^4 \times 5$

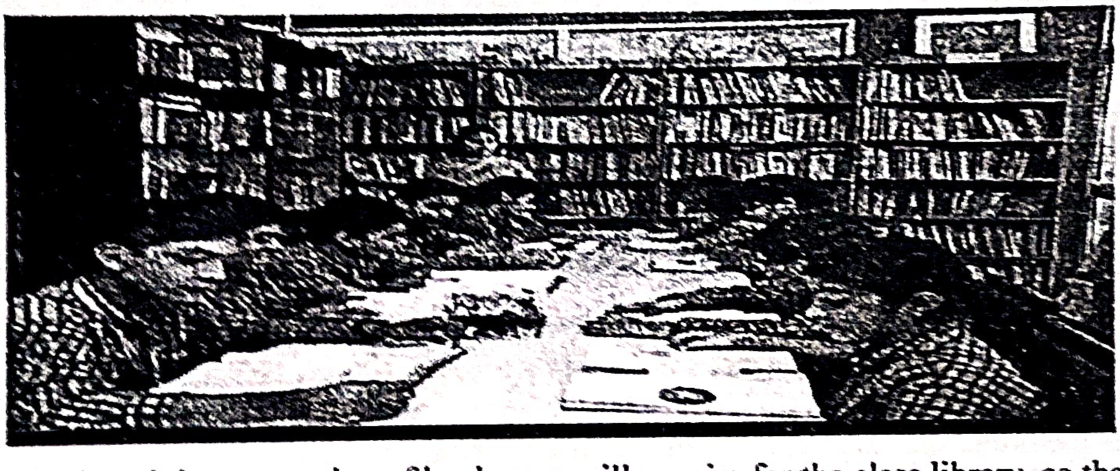
$720 = 2^4 \times 3^2 \times 5$
HCF $= 3^2 \times 5 = 45$ ml = Capacity of glass
No. of glasses filled from 1st vessel = 720/45 = 16
No. of glasses filled from 2nd vessel = 405/45 = 9
Total number of glasses = 25

CASE BASED QUESTIONS (4 MARKS QUESTIONS)

1. A garden consists of 135 rose plants planted in certain number of columns. There is another set of 225 marigold plants, which is to be planted in the same number of columns.

Read carefully the above paragraph and answer the following question
(i)Find the maximum number of columns in which they can be planted, also find the total number of plants.
(ii)Find the sum of exponents of the prime factors of the maximum number of columns in which they can be planted.
(iii)What is the total number of rows in which they can be planted.
Ans: (i) 45 and 360 (ii)3 (iii)8

2. To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B



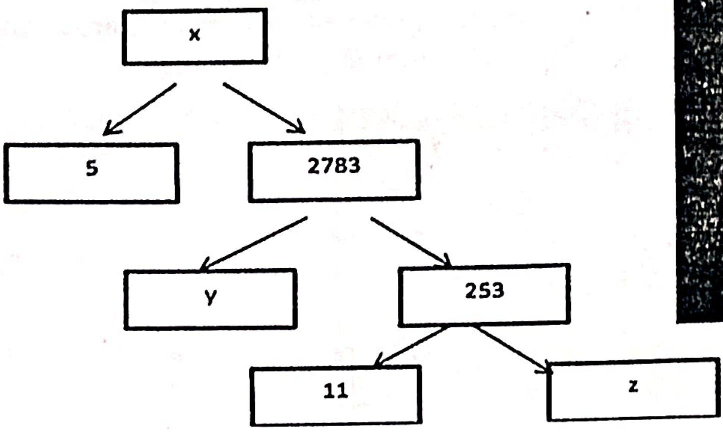
(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?
(ii)If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32, 36) is
(iii)Express 36 as a product of its primes.
Ans: (i) 288 (ii) 4 (iii) $2^2 \times 3^2$

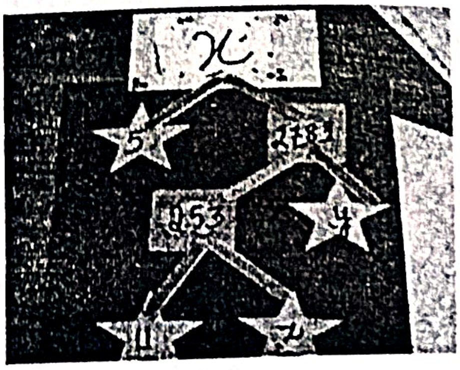
3. Raman considers eating nutritious food as an important part of his daily life. So on his birthday he decides to avoid junk food and plans to serve fruit to his friends. He has 60 bananas and 36 apples, which are to be distributed equally to his friends. Now answer the following:
(i) How many friends can he invite?
(ii) How many apples will each guest get?

(iii) Find the LCM of 60 and 36.

Ans: (i) 12 (ii) 3 (iii) 180

4.A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.





Observe the following factor tree and answer the following:

(i) What will be the value of x?

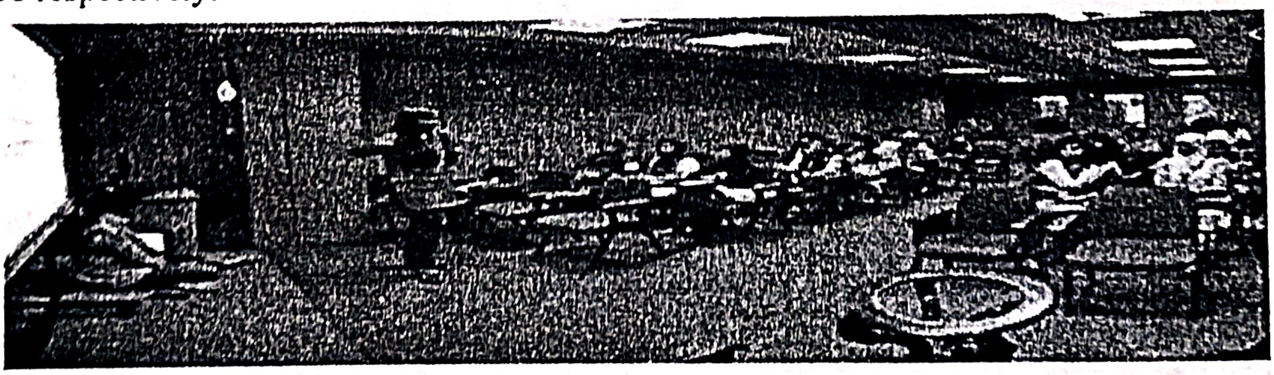
(ii) What will be the value of y?

(iii)What will be the value of z?

(iv)Find the prime factorisation of 13915 .

Ans: (i) 13915 (ii) 11 (iii) 23 (iv) $5\times11^2\times23$.

5.A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.



(i) In each room the same number of participants are to be seated and all of them being in the same subject, hence find the maximum number participants that can accommodated in each room.

(ii) What is the minimum number of rooms required during the event?

(iii) Find the LCM of 60, 84 and 108.

(iv) Find the product of HCF and LCM of 60,84 and 108

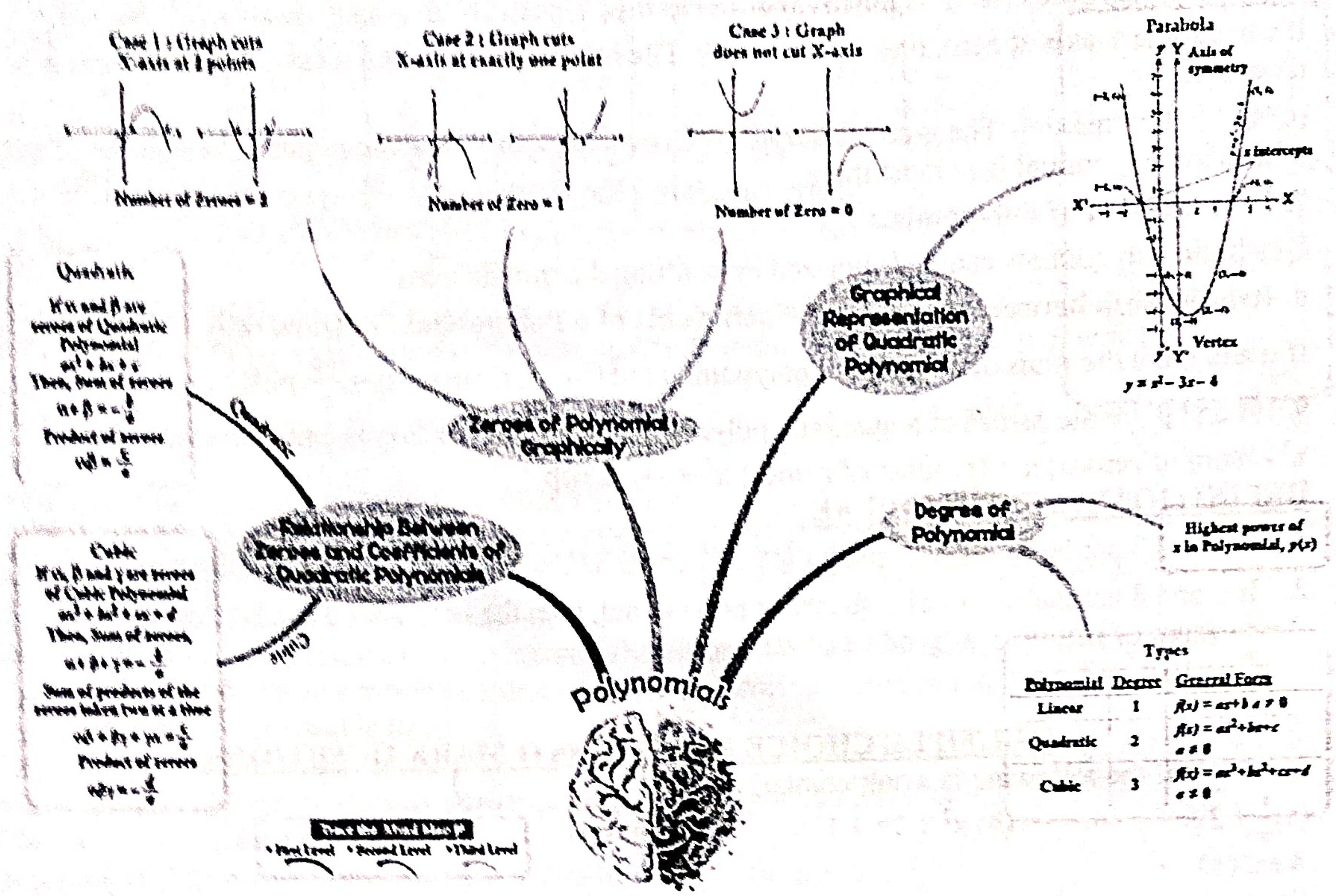
Ans: (i) 12 (ii) 21 (iii) 3780 (iv) 45360.

HIGHER ORDER THINKING QUESTION

1. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Ans: In this question we have to find the HCF of 144 and 90, which is 18.

CHAPTER-2
POLYNOMIALS
MIND MAPPING:



GIST/SUMMARY OF THE LESSON:

1.An algebraic expression is an expression made up of variables and constants along with mathematical operators. An algebraic expression is a sum of terms, which are considered to be building blocks for expressions.

2.Polynomial comes from the word 'Poly' (Meaning Many) and 'nomial' (in this case meaning Term)-so it means many terms. A polynomial is made up of terms that are only added, subtracted or multiplied. A polynomial is an algebraic expression in which the exponent on any variable is a whole number.

3.Degree of a Polynomial -For a polynomial in one variable – the highest exponent on the variable in a polynomial is the degree of the polynomial.

4.Types of Polynomials-Polynomials can be classified based on (a)Number of terms(b) Degree of the polynomial.

(a) Number of terms –Monomial-(one term) Example: $2x$, $6x^2$, $9xy$

Binomial – (two unlike terms) Example: $4x^2+x$, $5x+4$

Trinomial – (three unlike terms) Example: x^2+3x+4

(b) Degree- Linear Polynomial-(degree is one) Example: $2x+1$

Quadratic Polynomial-(degree is two) Example: $3x^2+8x+5$

Cubic Polynomial-(degree is three) Example: $2x^3+5x^2+9x+15$

5.Zeroes of a Polynomial -A zero of a polynomial $p(x)$ is the value of x for which the value of $p(x)$ is 0. If k is a zero of $p(x)$, then $p(k)=0$. Example: For $p(x)=x^2-3x+2$, when $x=1$, the value of $p(x)=0$. So, 1 is a zero of $p(x)$

6. Geometrical Representation and meaning of the zeroes of a Polynomial

(a)Linear Polynomial-The graph of a linear polynomial is a straight line. It cuts the X-axis at exactly one point. The number of zero of polynomial is one.

(b)Quadratic Polynomial-The graph of a quadratic polynomial is a parabola. It looks like a U, which either opens upwards (if 'a' is positive) or opens downwards (if 'a' is negative) in ax^2+bx+c

It can cut the x-axis at zero, one or two points. The number of zero of a quadratic polynomial is at most two.

(c) Cubic Polynomial- The graph is curve which cuts the x-axis at some points. The number of zero of a cubic polynomial is at most three.

7. Factorisation of Polynomials

Quadratic polynomials can be factorized by splitting the middle term.

8. Relationship between Zeroes and Coefficients of a Polynomial for Quadratic Polynomial:

If α and β are the roots of a quadratic polynomial ax^2+bx+c, then, $\alpha+\beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

9. If α and β are the zeroes of a quadratic polynomial, then the polynomial can be formed as –

x^2 - (sum of zeroes) x + (product of zeroes) $x^2-(\alpha+\beta)x+\alpha\beta$

DEFINITIONS AND FORMULAE:

1. If α and β are the roots of a quadratic polynomial ax^2+bx+c, then, $\alpha+\beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$
2. If α and β are the zeroes of a quadratic polynomial, then the polynomial can be formed as –
   x^2 - (sum of zeroes) x + (product of zeroes)
   $x^2-(\alpha+\beta)x+\alpha\beta$

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1. Which of the following is a polynomial?

(a) $\frac{1}{x}+2$ (b) x^2+2x+1 (c) $x^2+\frac{1}{x}+1$ (d) $x^{-2}+3x+2$

Ans: (b)

Solution: A polynomial cannot have variables in the denominator or negative powers.

2. What is the degree of the polynomial $5x^4+2x^3-x+7$?

(a) 3 (b) 4 (c) 5 (d) 2

Ans: (b)

Solution: The highest power of the variable is 4, so the degree is 4.

3. The value of the polynomial $f(x) = x^2-4x+4$ at $x = 2$ is:

(a) 0 (b) 4 (c) 2 (d) -2

Ans: (a)

Solution: $f(2) = (2)^2-4(2)+4 = 4-8+4 = 0$

4. If one zero of the polynomial $x^2+7x+12$ is -3, the other is:

(a) -4 (b) 4 (c) 3 (d) -2

Ans: (a)

Solution: $x^2+7x+12 = (x+3)(x+4) \Rightarrow$ Other zero is -4.

5. For the polynomial x^2-5x+6, the sum of the zeroes is:

(a) 5 (b) -5 (c) 6 (d) -6

Answer: (a)

Solution: Sum $= \frac{-b}{a} = \frac{-(-5)}{1} = 5$

6. If one zero of the polynomials (ax^2+bx+c) is the reciprocal of the other, and $(a \neq 0)$, then which of the following must be true?

(a) $b = c$ (b) $b^2 = 4ac$ (c) $c = a$ (d) $a = 0$

Ans: (c)

Solution: Let the zeroes be α and $\frac{1}{\alpha}$. Their product is 1. Hence, $\frac{c}{a} = 1 \Rightarrow c = a$

7. If the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ is factored completely, which of the following is NOT a zero?

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (d)

Solution: Factorization: $(x-1)(x-2)(x-3)$. So zeroes are 1, 2, 3. 4 is not a zero.

8. For what value of k is $2x^3 + 3x^2 - 2x + k$ divisible by $(x - 1)$?

(a) -2 (b) -3 (c) 1 (d) 2

Ans: (b)

Solution: Use Remainder Theorem: $f(1) = 0 \Rightarrow 2 + 3 - 2 + k = 0 \Rightarrow k = -3$

9. Which of the following quadratic polynomials has zeroes that are equal and real?

(a) $x^2 + 2x + 3$ (b) $x^2 + 4x + 4$ (c) $x^2 + 3x + 5$ (d) $x^2 - 2x + 5$

Ans: (b)

Solution: Discriminant $D = b^2 - 4ac = 16 - 16 = 0$, so real and equal roots.

10. If the sum and product of the zeroes of a quadratic polynomial are both equal to 5, which polynomial matches this condition?

(a) $x^2 - 5x + 5$ (b) $x^2 + 5x + 5$ (c) $x^2 - 10x + 25$ (d) $x^2 - 5x + 10$

Ans: (a)

Solution: For sum = product = 5, polynomial is $x^2 - 5x + 5$

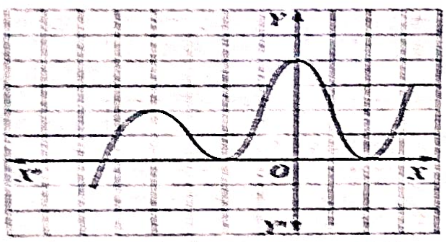
ASSERTION AND REASONING BASED QUESTIONS

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as:

(a)Both assertion and reason are true and reason is the correct explanation of assertion.
(b)Both assertion and reason are true but reason is not the correct explanation of assertion.
(c)Assertion is true but reason is false.
(d)Assertion is false but reason is true.

1. **Assertion(A):** The graph $y=f(x)$is shown in figure, for the polynomial f(x). The number of zeroes of (x) is 3.

Reason(R): The number of zero of the polynomial f(x) is the number of points of which f(x) cuts or touches the axes.



Ans: Graph cuts X axis at three points. So (a) is correct option.

2. **Assertion(A):** Both zeroes of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is ½.

Reason(R): Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$. is $\frac{-b}{a}$

Ans. (d)

3. **Assertion(A):** $x^2 + 4x + 5$ has two real zeroes.

Reason(R): A quadratic polynomial can have at the most two zeroes.

Ans: (d) $p(x) = 0 \Rightarrow x^2 + 4x + 5 = 0$ Discriminant, $D = b^2 - 4ac = -4 < 0$, therefore, no real zeroes are there.

4. **Assertion(A):** If the sum of the zeroes of the quadratic polynomial $x^2 - 2kx + 8$ is 2 then value of k is 1.

Reason(R): Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$

Ans : (a)

5. **Assertion(A):** $P(x = 4x^3 - x^2 + 5x^4 + 3$ is a polynomial of degree 3.

Reason(R): The highest power of variable is the degree of polynomial.
Ans: (d)

6. **Assertion(A) :** $x^3 + x$ has only one real zero.
Reason(R): A polynomial of nth degree must have n real zeroes.
Ans : (c)

7. **Assertion(A) :** Degree of zero of polynomial is not defined.
Reason(R): Degree of non zero constant polynomial is zero.
Ans: (b).

8. **Assertion(A):** If the product of the zeroes of polynomial $x^2 + 3x + k$ is -10, the value of k is -2
Reason(R): Sum of the zeroes of quadratic polynomial ax^2+bx+c is $\frac{-b}{a}$
Ans : (d)

9. **Assertion(A) :** $3-\sqrt{5}$ is one of the zero of a quadratic polynomial. Then the other zero is $3+\sqrt{5}$
Reason(R): Irrational zeroes always occur in pairs.
Ans : (a)

10. **Assertion(A) :** A quadratic polynomial whose sum of zeroes is 12 and product is 8 is $x^2-20x+96$
Reason(R): If α and β are zeroes of polynomial, then the polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$
Ans: (d)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

1. Find the value of the polynomial $p(x) = 5x - 4x^2 + 3$ at $x = -1$.
Solution: Given polynomial $p(x) = 5x - 4x^2 + 3$., Substitute $x = -1$ in p(x) we get $p(-1) = -6$

2. Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$.
Solution: Let $p(x) = x^2 - 2x - 8$. So, $x - 4 = 0$ or $x + 2 = 0$, $x = 4$ or $x = -2$

3. Find a quadratic polynomial whose sum and product of zeroes are $\frac{1}{4}$ and -1 respectively.
Solution: Let the zeroes be α and β. A quadratic polynomial is given by k[x^2 – (sum of zeroes)x + product of zeroes], where k is a non-zero constant, after substituting the values in the formula we get $4x^2 - x - 4$

4. If one zero of the polynomial $p(x) = (k - 1)x^2 + kx + 1$ is -3, then find the value of k.
Solution: Given $p(x) = (k - 1)x^2 + kx + 1$.
Since -3 is a zero of p(x), then $p(-3) = 0$. On putting the values we get $k = \frac{8}{6} = \frac{4}{3}$

5. Find the sum and product of the zeroes of the polynomial $2x^2 - 8x + 6$.
Solution: Let the polynomial be $p(x) = 2x^2 - 8x + 6$.
Comparing this with $ax^2 + bx + c$, we have $a = 2$, $b = -8$, $c = 6$.
Sum of zeroes $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-8)}{2} = \frac{8}{2} = 4$. & Product of zeroes $(\alpha\beta) = \frac{c}{a} = \frac{6}{3} = 3$.

6. If α and β are the zeroes of the polynomial $x^2 + 7x + 10$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$
Solution: For the polynomial $x^2 + 7x + 10$, $a = 1$, $b = 7$, $c = 10$.
Sum of zeroes $(\alpha + \beta) = \frac{-b}{a} = \frac{-7}{1} = -7$ & Product of zeroes $(\alpha\beta) = \frac{c}{a} = \frac{10}{1} = 10$.
$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{(\beta + \alpha)}{(\alpha\beta)} = \frac{(\alpha + \beta)}{(\alpha\beta)} = \frac{-7}{10}$

7. Check whether -2 is a zero of the polynomial $p(x) = x^3 + x^2 - 2x$.
Solution: Given $p(x) = x^3 + x^2 - 2x$.
Substitute $x = -2$,
$p(-2) = (-2)^3 + (-2)^2 - 2(-2) = 0$
Since $p(-2) = 0$, -2 is a zero of the polynomial.

8. If the sum of the zeroes of the quadratic polynomial $kx^2 - 3x + 5$ is 1, find the value of k.
Solution: Let the polynomial be $p(x) = kx^2 - 3x + 5$. Here, $a = k$, $b = -3$, $c = 5$.
Sum of zeroes $= \frac{-b}{a} = \frac{-(-3)}{k} = \frac{3}{k}$ & Given that the sum of zeroes is 1. So, $\frac{3}{k} = 1 \Rightarrow k = 3$.

9. If the product of the zeroes of the quadratic polynomial $x^2 - 6x + k$ is 4, find the value of k.

Solution: Let the polynomial be $p(x) = x^2 - 6x + k$. Here, $a = 1$, $b = -6$, $c = k$.
Product of zeroes $= \frac{c}{a} = \frac{k}{1} = k$. Given that the product of zeroes is 4. So, $k = 4$.

10: Write the quadratic polynomial whose zeroes are 3 and -2.

Solution: Let the zeroes be $\alpha = 3$ and $\beta = -2$.
Sum of zeroes $(\alpha + \beta) = 3 + (-2) = 1$ & Product of zeroes $(\alpha\beta) = 3 \times (-2) = -6$.
A quadratic polynomial is given by $k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$.
$p(x) = k[x^2 - (1)x + (-6)] = k[x^2 - x - 6]$, Choosing $k=1$, $p(x) = x^2 - x - 6$.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1. Find the zeroes of the polynomial $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients.

Solution: Let $p(s) = 4s^2 - 4s + 1$.

To find zeroes, set $p(s) = 0$, so, the zeroes are $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$.

Verification:

For $p(s) = 4s^2 - 4s + 1$, we have $a = 4$, $b = -4$, $c = 1$.

Sum of zeroes $(\alpha + \beta) = \frac{1}{2} + \frac{1}{2} = 1$. & From coefficients, $\frac{-b}{a} = \frac{-(-4)}{4} = \frac{4}{4} = 1$. So, $\alpha + \beta = \frac{-b}{a}$.

Product of zeroes $(\alpha\beta) = (\frac{1}{2}) \times (\frac{1}{2}) = \frac{1}{4}$. & From coefficients, $\frac{c}{a} = \frac{1}{4}$. So, $\alpha\beta = \frac{c}{a}$.

Hence, the relationship is verified.

2. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - 5x + 4$, find the value of $\alpha^2 + \beta^2$.

Solution: For $p(x) = x^2 - 5x + 4$, we have $a = 1$, $b = -5$, $c = 4$.

Sum of zeroes $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-5)}{1} = 5$. & Product of zeroes $(\alpha\beta) = \frac{c}{a} = \frac{4}{1} = 4$.

We know that $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$.

So, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (5)^2 - 2(4) = 17$.

3. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

Solution: Dividend $p(x) = 3x^3 + x^2 + 2x + 5$

Divisor $g(x) = x^2 + 2x + 1$ (arranging in standard form)

Quotient $q(x) = 3x - 5$

Remainder $r(x) = 9x + 10$

4. Write quadratic polynomial whose zeroes are $5\sqrt{3}$ and $2\sqrt{5}$

Solution: Let the zeroes be $\alpha = 5\sqrt{3}$ and $\beta = 2\sqrt{5}$.

$\alpha + \beta = 5\sqrt{3} + 2\sqrt{5}$. $\alpha\beta = (5\sqrt{3})(2\sqrt{5}) = 10\sqrt{15}$

Quadratic polynomial $= x^2 - (5\sqrt{3} + 2\sqrt{5}) + 10\sqrt{15}$

5. Find the quadratic polynomial whose zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$.

Solution: Let the zeroes be $\alpha = 2 + \sqrt{3}$ and $\beta = 2 - \sqrt{3}$.

Sum of zeroes $(\alpha + \beta) = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 2 + 2 = 4$.

Product of zeroes $(\alpha\beta) = (2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$.

The quadratic polynomial is $k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$.

$p(x) = k[x^2 - 4x + 1]$. Choosing $k=1$, $p(x) = x^2 - 4x + 1$.

Answer: The quadratic polynomial is $x^2 - 4x + 1$.

6. If $(x + a)$ is a factor of the polynomial $2x^2 + 2ax + 5x + 10$, find the value of 'a'.

Solution: If $(x + a)$ is a factor of $p(x) = 2x^2 + 2ax + 5x + 10$, then $p(-a) = 0$.

$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0 \Rightarrow a = 2$.

7. If α and β are the zeroes of the polynomial $2y^2 + 7y + 5$, find the value of $(\alpha + 1)(\beta + 1)$.

Solution: For the polynomial $2y^2 + 7y + 5$, we have $a = 2$, $b = 7$, $c = 5$.

Sum of zeroes $(\alpha + \beta) = \frac{-b}{a} = \frac{-(7)}{2}$. & Product of zeroes $(\alpha\beta) = \frac{c}{a} = \frac{5}{2}$.

$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1 = \alpha\beta + (\alpha + \beta) + 1 = \frac{5}{2} + \frac{-(7)}{2} + 1 = \frac{5-7}{2} + 1 = 0$.

8. Find all the zeroes of the polynomial $x^3 + 3x^2 - 2x - 6$, if one of its zeroes is $\sqrt{2}$.

Solution : Let $p(x) = x^3 + 3x^2 - 2x - 6$.

We can factor p(x) by grouping: $p(x) = x^2(x + 3) - 2(x + 3) = (x^2 - 2)(x + 3)$

To find the zeroes, set $p(x) = 0 \Rightarrow (x^2 - 2)(x + 3) = 0$.

The zeroes are $\sqrt{2}$, $-\sqrt{2}$, and -3.

Given one zero is $\sqrt{2}$.

9. What must be subtracted from $p(x) = x^3 - 6x^2 - 15x + 80$ so that the resulting polynomial is exactly divisible by $g(x) = x^2 + x - 12$?

Solution: Divide p(x) by g(x) to find the remainder. The remainder is what must be subtracted.

After dividing we get the remainder is $4x - 4$.

So, $4x - 4$ must be subtracted from p(x) for it to be exactly divisible by g(x).

10. Form a quadratic polynomial one of whose zero is $2 + \sqrt{5}$ and the sum of zeroes is 4.

Solution: Let the zeroes be α and β.

Given one zero $\alpha = 2 + \sqrt{5}$ & sum of zeroes $(\alpha + \beta) = 4$.

So, $(2 + \sqrt{5}) + \beta = 4 \Rightarrow \beta = 2 - \sqrt{5}$.

Now, find the product of zeroes $(\alpha\beta)$:

$\alpha\beta = (2 + \sqrt{5})(2 - \sqrt{5}) = 2^2 - (\sqrt{5})^2 = 4 - 5 = -1$.

The quadratic polynomial is $k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$.

$p(x) = k[x^2 - (4)x + (-1)] = k[x^2 - 4x - 1]$.

Choosing k=1, $p(x) = x^2 - 4x - 1$.

LONG ANSWER TYPE QUESTIONS (5 MARK QUESTIONS)

1. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Solution: Given zeroes are $\alpha = \sqrt{\frac{5}{3}}$ and $\beta = -\sqrt{\frac{5}{3}}$.

The factors are $(x - \sqrt{\frac{5}{3}})$ and $(x + \sqrt{\frac{5}{3}})$.

Their product is $(x - \sqrt{\frac{5}{3}})(x + \sqrt{\frac{5}{3}}) = x^2 - (\sqrt{\frac{5}{3}})^2 = x^2 - \frac{5}{3}$

So, $(x^2 - \frac{5}{3})$ is a factor of the given polynomial.

We can write this as $(\frac{1}{3})(3x^2 - 5)$. So, $(3x^2 - 5)$ is also a factor.

Divide $3x^4 + 6x^3 - 2x^2 - 10x - 5$ by $3x^2 - 5$:

The quotient is $x^2 + 2x + 1$.

To find other zeroes, set the quotient to zero:

$x^2 + 2x + 1 = 0 \Rightarrow x = -1$

So, the other two zeroes are -1 and -1.

2. Find the values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

Solution: If $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$, the remainder must be zero.

Divide $x^4 + x^3 + 8x^2 + ax + b$ by $x^2 + 1$:

The remainder is (a -1)x + (b -7)
For exact divisibility, the remainder must be 0
So, (a -1)x + (b -7) = 0.x + 0
Comparing coefficients: a = 1 & b = 7

3. What must be added to the polynomial $p(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

Solution: Divide p(x) by $x^2 + 2x - 3$ to find the remainder.
The remainder is $-x + 2$.
Let R(x) be what must be added.
p(x) + R(x) should be divisible by $x^2 + 2x - 3$.
This means the remainder when p(x) + R(x) is divided by $x^2 + 2x - 3$ is 0.
The remainder when p(x) is divided by $x^2 + 2x - 3$ is $(-x + 2)$.
So, we must add $-(-x + 2)$ to p(x).
$-(-x + 2) = x - 2$.
Therefore, $x - 2$ must be added.

4. Verify the division algorithm for the polynomials $p(x) = x^4 - 5x + 6$ and $g(x) = 2 - x^2$.

Solution: $p(x) = x^4 - 5x + 6$
$g(x) = -x^2 + 2$ (writing in standard form)
Divide p(x) by g(x): Quotient $q(x) = -x^2 - 2$ & Remainder $r(x) = -5x + 10$
Division Algorithm: $p(x) = g(x) \times q(x) + r(x)$
RHS $= (-x^2 + 2)(-x^2 - 2) + (-5x + 10)$
$= [(-x^2)(-x^2 - 2) + 2(-x^2 - 2)] - 5x + 10 = x^4 - 5x + 6$
LHS $= p(x) = x^4 - 5x + 6$
Since LHS = RHS, the division algorithm is verified.

5. If α, β are the zeroes of the polynomial $kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, find the value of k.

Solution: For the polynomial $kx^2 + 4x + 4$: a = k, b = 4, c = 4.
Sum of zeroes $(\alpha + \beta) = \frac{-b}{a} = \frac{-4}{k}$ & Product of zeroes $(\alpha\beta) = \frac{c}{a} = \frac{4}{k}$.
We know that $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.
Given $\alpha^2 + \beta^2 = 24$.
So, $24 = (\frac{-4}{k})^2 - 2(\frac{4}{k})$
$24 = \frac{16}{k^2} - \frac{8}{k}$
$24k^2 = 16 - 8k$ [Multiply by k^2 (assuming $k \neq 0$)]
$24k^2 + 8k - 16 = 0$
$(3k - 2)(k + 1) = 0$
So, $3k - 2 = 0 \Rightarrow k = \frac{2}{3}$ (or), $k + 1 = 0 \Rightarrow k = -1$.

CASE BASED QUESTIONS

1. Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial.

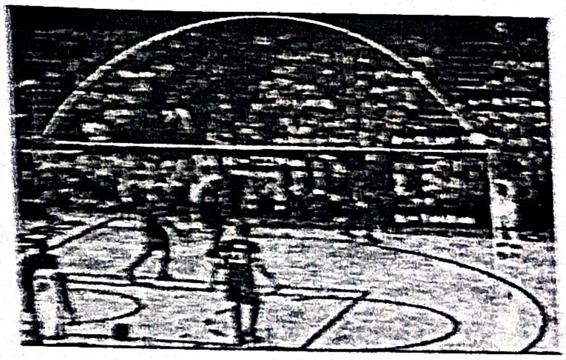


Fig -1

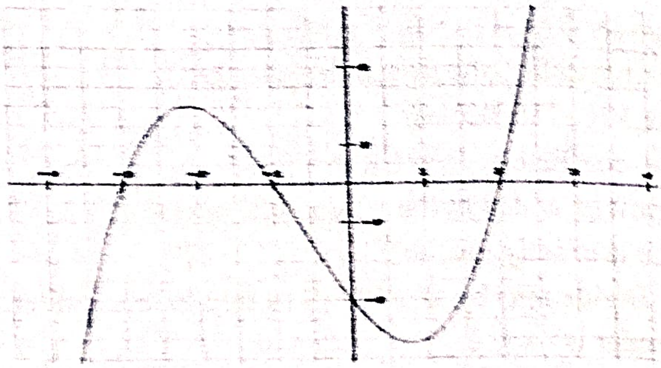


Fig - 2

(i) What is the shape of the path traced in Fig 1?

Ans: Parabola

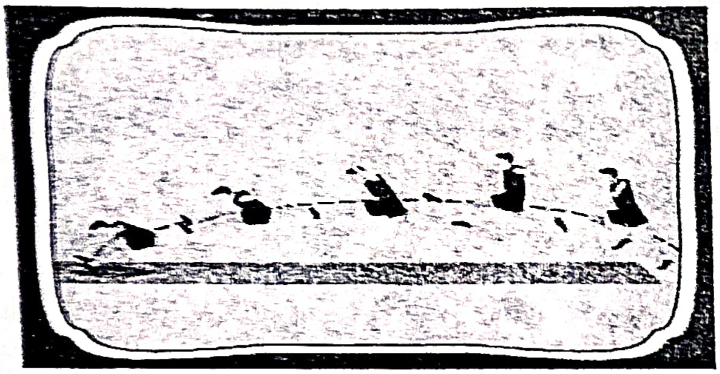
(ii) When the parabola is downwards, then what you can say about value of 'a'?

Ans : a<0

(iii) In Fig 2 how many zeroes are there and what are they?

Ans: Three, -3, -1 and 2

2. Observe the position of athletic taking long jump. He used to follow a particular type of path. In the figure we can observe the path followed by an athlete.



(i) What is the name of the path formed by different positions of the athlete.

Ans : Parabola

(ii) In the above case, if the quadratic polynomial is represented by $ax^2 + bx + c$, then what can you say about the value of 'a'?

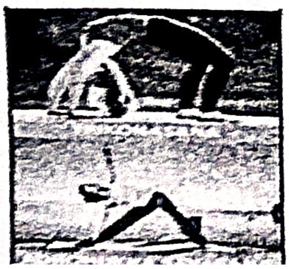
Ans :As the parabola open downwards, the value of 'a' will be less than 0. (a < 0)

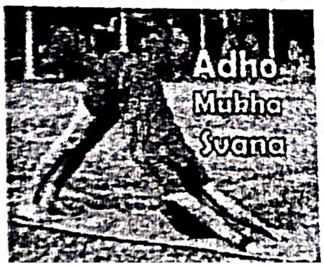
(iii) If the sum and product of the zeroes of the quadratic polynomial $ax^2 + bx + c$ are equal. What is the relation between 'b' and 'c'?

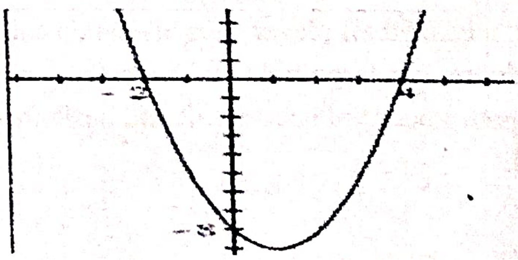
Ans: Given polynomial is $ax^2 + bx + c$. Sum of the zeroes $= \frac{-b}{a}$, Product of the zeroes $= \frac{c}{a}$

According to question, $\frac{-b}{a} = \frac{c}{a} \Rightarrow -b = c \Rightarrow b + c = 0$

3. An asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial.







(i) What is the shape of figure shown?

Ans : Parabola

(ii) When the parabola is upwards, then what you can say about value of 'a'?

Ans : a>0

(iii) In the graph how many zeroes are there and what are they?

Ans : two zeroes, -2,4

HIGHER ORDER THINKING SKILL QUESTIONS

1. If one zero of the polynomial $p(x) = ax^2 + bx + c$ is twice the other, prove that $2a\alpha^2 + b\alpha + c = 0$ and hence find a relation between a, b, and c.

Solution: Let one zero be α, so the other is 2α.

Sum of zeros $= \alpha + 2\alpha = 3\alpha = \frac{-b}{a} \Rightarrow \alpha = \frac{-b}{3a}$

Product $= \alpha \cdot 2\alpha = 2\alpha^2 = \frac{c}{a} \Rightarrow \alpha^2 = \frac{c}{2a}$

Now, $2a\alpha^2 + b\alpha + c = 2a\cdot(\frac{c}{2a}) + b\cdot(\frac{-b}{3a}) + c = c - \frac{b^2}{3a} + c$

$\Rightarrow 2c - \frac{b^2}{3a} = 0 \Rightarrow 6ac = b^2$

2. A quadratic polynomial has its zeros as reciprocals of each other. If its leading coefficient is 3, find the polynomial.

Solution: Let the zeros be α and $\frac{1}{\alpha}$

Sum $= \alpha + \frac{1}{\alpha}$, Product $= 1$

Polynomial: $3x^2 - 3(\alpha + \frac{1}{\alpha})x + 3$

Assuming $\alpha = 2 \Rightarrow$ Sum $= 2.5 \Rightarrow$ Polynomial: $6x^2 - 15x + 6$

3. A quadratic polynomial f(x) has the property that $f(1) = f(-1)$. Show that the coefficient of x must be zero.

Solution: Let $f(x) = ax^2 + bx + c$

$f(1) = a + b + c$, $f(-1) = a - b + c$

$f(1) = f(-1) \Rightarrow a + b + c = a - b + c \Rightarrow 2b = 0 \Rightarrow b = 0$

4. Form a quadratic polynomial whose sum of the zeros is equal to their product and both zeros are rational numbers.

Solution: Let $\alpha = \beta = r \Rightarrow$ Sum $= 2r$, Product $= r^2 \Rightarrow 2r = r^2 \Rightarrow r(r - 2) = 0 \Rightarrow r = 0$ or 2

If $r = 2 \Rightarrow$ Polynomial: $(x - 2)^2 = x^2 - 4x + 4$

5. A polynomial p(x) leaves the same remainder when divided by x - 1 and x + 1. Show that the coefficient of the odd powers of x in p(x) is zero.

Solution: $p(1) = p(-1)$

Let $p(x) = a_0 + a_1x + a_2x^2 + \ldots$

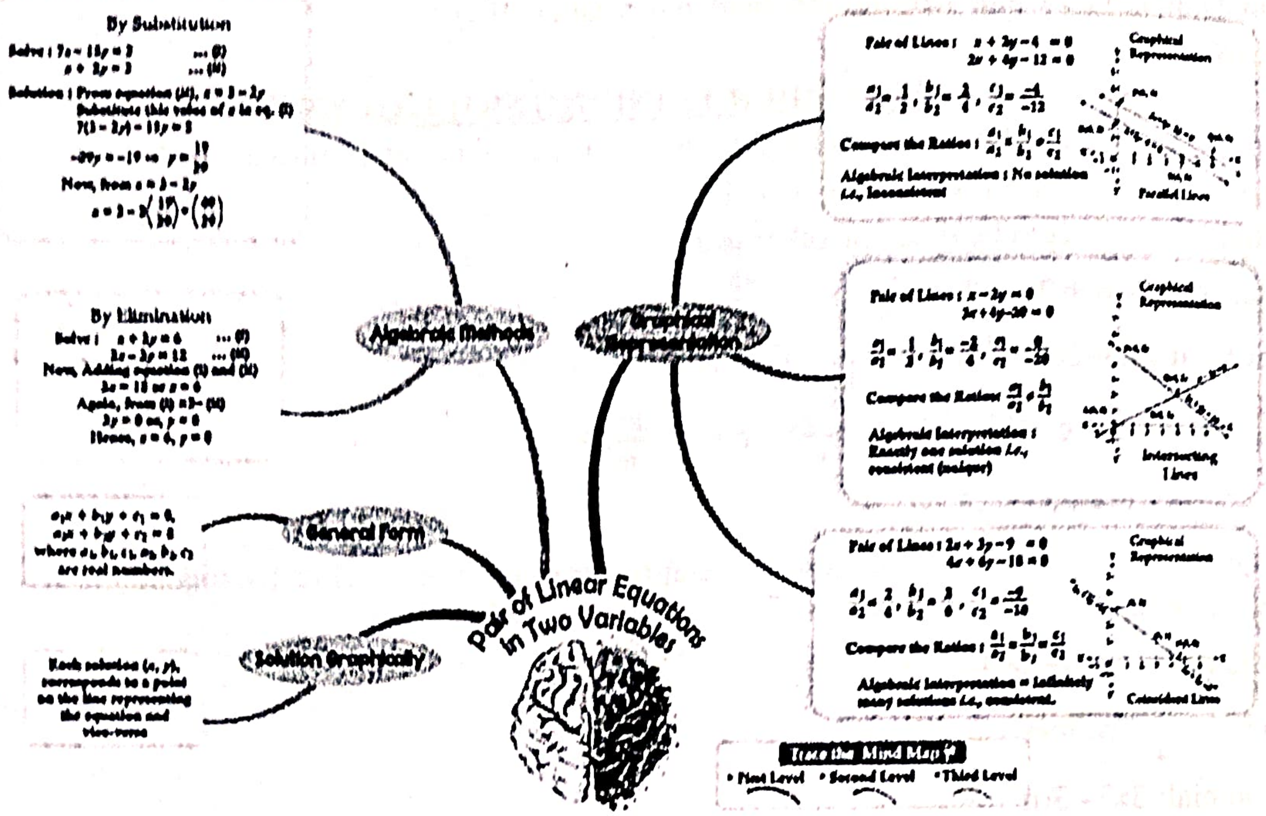
$p(1) = a_0 + a_1 + a_2 + \ldots$, $p(-1) = a_0 - a_1 + a_2 - \ldots$

If $p(1) = p(-1)$

$\Rightarrow 2a_1 + 2a_3 + \ldots = 0$

$\Rightarrow a_1 = a_3 = \ldots = 0$

CHAPTER-3
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES
MIND MAPPING



Gist of the lesson:

1. Definition of the terms
2. Solving of linear equations in two variables --(a) Graphical Method (b) Algebraic Method
3. Algebraic Method- (a) Substitution (b) Elimination

Definitions and Formulae:

1)Linear Equation in Two Variables- An equation which can be put in the form ax + by + c = 0 where a,b,c are Real Numbers & a,b are not both zero, is called a Linear Equation in Two Variables x & y.
Example:- 2x+5y-6=0
[a = 2, b = 5,c= -6]

2) General Form for a Pair of Linear Equations in Two Variables x & y-
$a_1x + b_1y + c_1 = 0$ [a_1, b_1, c_1 are Real Numbers & a_1, b_1 are not both zero $a_2x+b_2y + c_2 = 0$
a_2,b_2,c_2 are Real Numbers & a_2, b_2 are not both zero]

3) Method of Finding the Solution for a Pair of Linear Equations in Two Variables x & y-
i)Graphical Method
ii)Algebraic Method
 a)Substitution Method
 b) Elimination Method

4)Graphical Method
Plot the graph of the first equation and then graph of the second equation on the same rectangular coordinate system. The following three cases may arise.
Case 1 - If the lines intersect at a point, then the given system has a unique Solution given by the coordinates of the point of intersection.
Case 2 - If the lines are coincident, then the system is consistent and has infinitely many Solutions. In this case, every Solution of one of the equations is a Solution of the system.
Case 3 - If the lines are parallel, then the given system of equations is inconsistent i.e., it has no Solution.

5)Substitution Method

Step 1: Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.

Step 2: Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved.

Step 3: Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

6)ELIMINATION METHOD

Step 1: First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2: Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to Step 3.

If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many Solutions.

If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no Solution, i.e., it is inconsistent.

Step 3: Solve the equation in one variable (x or y) so obtained to get its value.

Step 4: Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS).

Q1. The **Solution** of the equations $x - y = 2$ and $x + y = 4$ is:

(a) 3 and 1 (b) 4 and 3 (c) 5 and 1 (d) -1 and -3

Solution: The system of equations $x - y = 2$ and $x + y = 4$ is solved using elimination. Adding the equations gives $2x = 6$, so $x = 3$. Substituting into the second equation gives $3 + y = 4$, so $y = 1$. The Solution is $x = 3$, $y = 1$.

Ans: (a) 3 and 1

Q2. If the lines $3x + 2ky - 2 = 0$ and $2x + 5y + 1 = 0$ are parallel, then what is the value of k?

(a) $\frac{4}{15}$ (b) $\frac{15}{4}$ (c) $\frac{4}{5}$ (d) $\frac{5}{4}$

Solution: To make the lines $3x + 2ky - 2 = 0$ and $2x + 5y + 1 = 0$ parallel, we need $\frac{3}{2} = \frac{2k}{5}$. Solving for k gives $k = \frac{15}{4}$. This value satisfies the parallel lines condition because $\frac{2k}{5} \neq \frac{-2}{1}$.

Ans: (b) $\frac{15}{4}$

Q3. The pair of equations $3x - 5y = 7$ and $-6x + 10y = 7$ have

(a) a unique Solution (b) infinitely many Solutions

(c) no Solution (d) two Solutions

Solution: $3x - 5y = 7$ and $-6x + 10y = 7$ have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, thus, no Solution.

Ans: (c) no **Solution**

Q4. If a pair of linear equations is consistent, then the lines will be

(a) always coincident (b) parallel

(c) always intersecting (d) intersecting or coincident

Solution: Consistent linear equations have at least one **Solution**; lines can be intersecting or coincident. **Ans:** (d) intersecting or coincident

Q5. The pair of equations $5x + 7y = 5$ and $2x - 3y = 7$ are

(a) consistent (b) inconsistent (c) coincident (d) none of these

Solution: $5x + 7y = 5$ and $2x - 3y = 7$ have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so consistent

Ans: (a) consistent

Q6. Which of the following pair of equations are dependent?

(a) $2x + 3y = 9$ and $4x + 6y = 18$ (b) $x + y = 2$ and $2x - y = 4$

(c) $5x + y = 10$ and $2x + y = 6$ (d) $x - y = 0$ and $x + y = 4$

Solution: Dependent equations are those that represent the same line (coincident lines), meaning they have infinitely many Solutions. The condition is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Dependent equations are coincident lines. $2x + 3y = 9$ and $4x + 6y = 18$ have $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, hence (a) is dependent.

Ans:(a) $2x + 3y = 9$ and $4x + 6y = 18$

Q7. The value of c for which the pair $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many Solutions is:

(a) 3 (b) -3 (c) -12 (d) No value

Solution: For infinitely many solutions, we need $\frac{c}{6}=\frac{-1}{-2}=\frac{-2}{-3}$, or $\frac{c}{6}=\frac{1}{2}=\frac{2}{3}$. Since $\frac{1}{2}\neq\frac{2}{3}$, no value of c satisfies the condition $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, so, there is no value of c.

Ans: (d) No value

Q8. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to the denominator. The fraction is:

(a) $\frac{3}{12}$ (b) $\frac{4}{12}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$

Solution: Let the fraction be represented as $\frac{x}{y}$. The problem provides two equations based on alterations to the numerator and denominator: $\frac{x-1}{y}=\frac{1}{3}$ and $\frac{x}{y+8}=\frac{1}{4}$.

Solving this system of equations leads to $x = 5$ and $y = 12$. Therefore, the original fraction is $\frac{5}{12}$.

Ans: (c) $\frac{5}{12}$

Q9. The graph of equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ represents:

(a) intersecting lines (b) parallel lines (c) coincident lines (d) none

Solution: $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ have $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$,, therefore (b) parallel lines.

Ans: (b) parallel lines

Q10. Which of the following is a condition for unique solution of a pair of linear equations?

(a) $\frac{a_1}{a_2}=\frac{b_1}{b_2}$ (b) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, (c) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ (d) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Solution:

The condition for a unique Solution of linear equations is (c) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$.

Ans: (c)

ASSERTION AND REASON QUESTIONS

Directions: In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

1.Assertion (A): A pair of linear equations has no Solution if it is represented by intersecting lines.
Reason (R): Intersecting lines represent a pair of equations having a unique **Solution.**
Ans: (d)

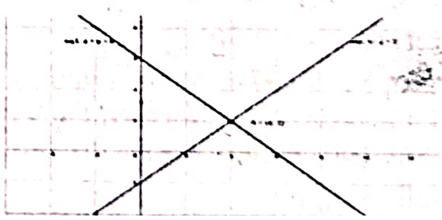
2.Assertion (A): If the pair of equations has the same **Solution**, then it is called consistent.
Reason (R): Consistent system has at least one **Solution.**
Ans: (a)

3.Assertion (A): The pair of equations $2x + 3y = 6$ and $4x + 6y = 10$ are consistent.
Reason (R): They represent parallel lines.
Ans: (d)

4.**Assertion (A):** The graph of a pair of linear equations is a pair of parallel lines if there is no solution.
Reason (R): Parallel lines never intersect and hence are inconsistent.
Ans: (a)
5.**Assertion (A):** If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, the system is inconsistent.
Reason (R): Such a system represents parallel lines.
Ans: (a)
6.**Assertion (A):** The equations $x + 2y = 3$ and $2x + 4y = 6$ represent intersecting lines.
Reason (R): Their slopes are same.
Ans: (d)
7.**Assertion (A):** The pair $2x + 3y = 8$ and $4x + 6y = 10$ has a unique solution.
Reason (R): Lines with different slopes intersect at one point.
Ans: (d)
8.**Assertion (A):** The system of equations $x + y = 5$ and $2x + 2y = 10$ has infinitely many solutions.
Reason (R): Both equations represent the same line.
Ans: (a)
9.**Assertion (A):** If the graphical representation of a system of equations results in two parallel lines, the system is inconsistent.
Reason (R): In an inconsistent system, there is no point common to both lines.
Ans: (a)
10.**Assertion (A):** A system of equations with infinitely many **Solutions** must be inconsistent.
Reason (R): Coincident lines have no unique solution.
Ans: (d)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

1. Solve the pair of linear equations:$3x - y = 3$ and $9x - 3y = 9$
Solution: The equation $9x - 3y = 9$ is simply a multiple of $3x - y = 3$. They represent the same line on a graph, thus the system has infinitely many Solutions because every point on the line is a Solution.
2. Solve the following pair of equations: $s + t = 15$, $2s - 3t = 5$
Solution: Using substitution on the system $s + t = 15$ and $2s - 3t = 5$, we isolate 's' in the first equation ($s = 15-t$) and substitute it into the second. Solving the resulting equation for 't' yields $t = 5$. Plugging $t = 5$ back into $s = 15-t$ gives $s = 10$. Therefore, the Solution is $s = 10$ and $t = 5$.
3. Find the value of k so that the pair $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.
Solution: For linear equations to have a unique solution, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$. Applying this to $x + 2y = 5$ and $3x + ky + 15 = 0$ requires that $\frac{1}{3}\neq\frac{2}{k}$.
Solving this inequality shows that k cannot be equal to 6 for a unique solution to exist. $k \neq 6$
4. Find whether the equations are consistent or inconsistent: $5x + 7y = 5$, $2x - 3y = 7$
Solution: The system $5x + 7y = 5$ and $2x - 3y = 7$ has) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$, so the lines must intersect in one point and have exactly one **Solution**, meaning it is consistent with a unique **Solution**.
Consistent (unique **Solution**)
5. Half the perimeter of a rectangular garden is 36 m. If the length is 4 m more than the width, form the pair of equations.
Solution: We represent length and width of a rectangular garden as 'x' and 'y' respectively. Half the perimeter is 'x + y', which equals 36. Also, the length (x) is 4 more than the width (y), which translates into the expression $x = y + 4$. Thus the equations are, $x+y=36$, $x = y + 4$.
6. Determine graphically the solution of:
$x + y = 6$, $x - y = 2$
Solution: $x = 4$, $y = 2$ (The graphical solution would be the graph showing the intersection at (4,2)).



Q7. Solve: $x + 2y - 4 = 0$, $2x + 4y - 12 = 0$

Solution: The equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ have proportional x and y coefficients ($\frac{a_1}{a_2} = \frac{b_1}{b_2}$), but different constant ratios ($\frac{c_1}{c_2}$). Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, they represent parallel lines that never intersect. Therefore, the system has no **Solution** and is considered inconsistent.

Q8. Represent the following situation algebraically:

"The cost of 2 kg apples and 1 kg grapes is Rs.160. The cost of 4 kg apples and 2 kg grapes is Rs300."

Solution: $2x + y = 160$, $4x + 2y = 300$ (Cost Problem): Letting 'x' be the cost of 1 kg of apples and 'y' be the cost of 1 kg of grapes, the provided information directly translates into the linear equations: $2x + y = 160$ and $4x + 2y = 300$.

9. Check if the equations are dependent: $2x + 3y = 9$ and $4x + 6y = 18$

Solution: The equation $4x + 6y = 18$ is just double the equation $2x + 3y = 9$. These two equations represent the same line, therefore the equations are dependent and have infinitely many solutions.

10. A number consists of two digits. The digit at the tens place is 3 more than the unit's digit. The sum of the digits is 9. Find the number.

Solution: Let 'x' be the units digit and 'y' be the tens digit. The digits relationship forms two equations:

$y = x+3$ and $x+y = 9$.

Solving these simultaneously gives $x=3$ and $y=6$, so the number is $10(y)+x = 10(6)+3 = 63$.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1. Find the value of k for which the system: $x + 2y = 5$, $3x + ky + 15=0$ has a unique solution.

Solution: For $x + 2y = 5$, $3x + ky + 15 = 0$ to have a unique Solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. This translates to $\frac{1}{3} \neq \frac{2}{k}$.

Solving for k gives $k \neq 6$

2. A fraction becomes $\frac{5}{7}$ if 2 is added to both numerator and denominator. If 1 is subtracted from both, it becomes $\frac{1}{2}$. Find the fraction.

Solution: Let the fraction be $\frac{x}{y}$. We have $\frac{(x+2)}{(y+2)} = \frac{5}{7}$ and $\frac{(x-1)}{(y-1)} = \frac{1}{2}$.

Solving the equations $7x - 5y = -4$ and $2x - y = 1$ gives $x = 3$ and $y = 5$. The fraction is $\frac{3}{5}$

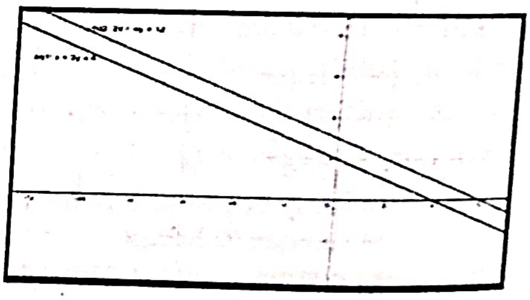
3. Solve the pair: $49x + 51y = 499$, $51x + 49y = 501$

Solution: Adding and subtracting the equations $49x + 51y = 499$ and $51x + 49y = 501$ yields $x + y = 10$ and $-x + y = -1$. Solving gives $x = \frac{11}{2}$, $y = \frac{9}{2}$.

4. Graphically solve the pair: $x + 2y = 4$, $2x + 4y = 12$

Solution: Graphing $x + 2y = 4$ and $2x + 4y = 12$ shows parallel lines, as they have the same slope.

Since they don't intersect, there is no solution.



5. Three years hence, a father's age will be three times that of his son. Five years ago, the father was seven times the son's age. Find their present ages.

Solution: Let father's age be 'x' and son's 'y'.

Three years hence: $x+3 = 3(y+3)$.

Five years ago: $x-5 = 7(y-5)$.

Solving $x-3y = 6$ and $x-7y = -30$ gives $y = 9$ and $x = 33$.

∴Father's present age = 33 years, Son's present age = 9 years.

6. Solve by substitution method: $7x - 15y = 2$, $x + 2y = 3$

Solution: Solving for x=3-2y in the second equation and using in the first equation you get: $x=\frac{49}{29}$, $y=\frac{19}{29}$

$\therefore x=\frac{49}{29}, y=\frac{19}{29}$ is the solution

7. Solve the pair: $x+y=6, x-y=4$

Solution: Adding $x+y=6$ and $x-y=4$ yields $2x=10$, so $x=5$. Substituting into $x+y=6$, we get $y=1$.

$\therefore x=5, y=1$ is the solution

8. The cost of 2 pencils and 3 pens is Rs11. The cost of 1 pencil and 2 pens is Rs7. Find the cost of each item.

Solution: Let pencil cost Rs x and pen Rs y. The system is $2x+3y=11, x+2y=7$.
Solving by elimination/substitution yields $x=1$ and $y=3$.
$\therefore$ Cost of one pencil = Rs 1, Cost of one pen = Rs 3.

9. Form the pair of equations: "The sum of two numbers is 25. One number is 3 less than double the other."

Solution: "The sum of two numbers is 25," so $x+y=25$.
"One number is 3 less than double the other" gives $x=2y-3$ or $x-2y=-3$.

10. Solve the following pair using substitution: $3x+2y=16, x=y+2$

Solution: Given $3x+2y=16, x=y+2$, substitute the second equation into the first.
This gives $3(y+2)+2y=16$, so $5y=10$. Therefore, $y=2$ and $x=4$.

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1. On reversing the digits of a two-digit number, the number obtained is 9 less than three times the original number. If the difference between the numbers is 45, find the original number.

Solution: Units digit = x, Tens digit = y. Original: $10y+x$. Reversed: $10x+y$.
Condition 1: $10x+y=3(10y+x)-9 \Rightarrow 7x-29y=-9$ (Eq. 1)
Condition 2: $|(10y+x)-(10x+y)|=45$.
Case 1: $10x+y-(10y+x)=45 \Rightarrow 9x-9y=45 \Rightarrow x-y=5$ (Eq. 2a)
Case 2: $10y+x-(10x+y)=45 \Rightarrow 9y-9x=45 \Rightarrow y-x=5$ (Eq. 2b)
Using Eq. 2a: $x=y+5$. Substitute into Eq. 1:, $7(y+5)-29y=-9 \Rightarrow -22y=44 \Rightarrow y=2$.
$x=2+5=7$.
$\therefore$ Original number $=10(2)+7=27$.

2. A shopkeeper buys 2 pencils and 3 pens for Rs.11. Sumeet buys 1 pencil and 2 pens for Rs.7. Find the cost of 1 pencil and 1 pen.

Solution: Pencil cost = x, Pen cost = y. Condition 1: $2x+3y=11$ (Eq. 1)
Condition 2: $x+2y=7$ (Eq. 2) Multiply Eq. 2 by 2: $2x+4y=14$ (Eq. 3)
Eq. 3 - Eq. 1: $y=3$. Substitute $y=3$ into Eq. 2: $x+2(3)=7 \Rightarrow x=1$.
$\therefore$Pencil = Rs1, Pen = Rs 3. Cost of 1 pencil and 1 pen = Rs 4.

3. A boat takes 2 hours to travel 20 km downstream and 4 hours to travel the same distance upstream. Find the speed of boat and speed of stream.

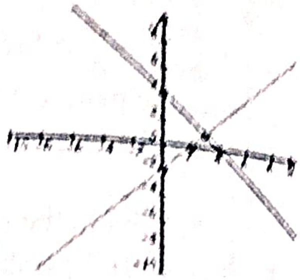
Solution: Boat speed = x, Stream speed = y. Downstream: $20=(x+y)2 \Rightarrow x+y=10$ (Eq. 1).
Upstream: $20=(x-y)4 \Rightarrow x-y=5$ (Eq. 2)
Eq. 1 + Eq. 2: $2x=15 \Rightarrow x=7.5$,
Substitute $x=7.5$ into Eq. 1: $7.5+y=10 \Rightarrow y=2.5$.
$\therefore$ Boat speed = 7.5 km/h, Stream speed = 2.5 km/h

4. Solve the pair: $2x+3y=12, x-y=1$

Solution: Eq. 1: $2x+3y=12$, Eq. 2: $x-y=1 \Rightarrow x=y+1$
Substitute $x=y+1$ into Eq. 1: $2(y+1)+3y=12 \Rightarrow 5y+2=12 \Rightarrow 5y=10 \Rightarrow y=2$
$x=2+1=3$.
$x=3, y=2$ is the solution.

5. The sum of the digits of a two-digit number is 9. If 27 is subtracted from the number, the digits

Solution: Tens digit = x, Units digit = y, original number: 10x + y.
Condition 1: x + y=9 (Eq.1), Condition 2: (10x + y) - 27=10y + x => 9x - 9y=27 => x - y = 3 (Eq.2),
Eq. 1 + Eq. 2: 2x = 12 => x = 6, Substitute x = 6 into Eq. 1: 6 + y = 9 => y = 3
∴ Original number: 10(6) + 3 = 63
6. Solve graphically: x + y = 4, x − y = 2
Solution: x= 3, y =1.

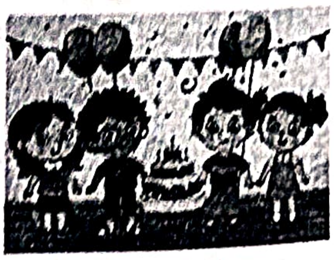


The graph would show two lines intersecting at the point (3,1)).

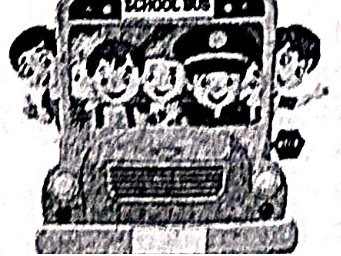
7. The cost of 4 oranges and 3 mangoes is Rs.30. The cost of 2 oranges and 4 mangoes is Rs 28. Find cost of one orange and one mango.
Solution: Orange cost = x, Mango cost = y.
Condition 1: 4x + 3y = 30 (Eq. 1), Condition 2: 2x + 4y = 28 (Eq. 2)
Multiply Eq. 2 by 2: 4x + 8y = 56 (Eq. 3), Eq. 3 - Eq. 1: 5y = 26 => y = 5.2
Substitute y = 5.2 into Eq. 2: 2x + 4(5.2) = 28 => 2x + 20.8 = 28 => 2x = 7.2 => x = 3.6
∴ Orange cost = Rs 3.60, Mango cost = Rs 5.20
Q8. Solve: $\frac{1}{x}+\frac{1}{y}=5, \quad \frac{1}{x}-\frac{1}{y}=1$
Solution: $a=\frac{1}{x}$, $b=\frac{1}{y}$,, Eq. 1: a + b = 5, Eq. 2: a - b = 1, Add equations: 2a = 6 => a = 3
Substitute into Eq. 1: 3 + b = 5 => b = 2, a = 3 => $x=\frac{1}{3}$, b = 2 => $y=\frac{1}{2}$
9. A father is three times as old as his son. After 5 years, he will be twice as old. Find their ages.
Solution: Son's age = x, Father's age = 3x, After 5 years: Son: x + 5, Father: 3x + 5
3x + 5 = 2(x + 5) => 3x + 5 = 2x + 10 => x = 5,
∴Son' age = 5yrs , Father's age = 3(5) = 15 yrs
10. Represent graphically and solve: x + y = 10, x − y = 2
Solution: Eq. 1: x + y = 10, Eq. 2: x - y = 2 , Add equations: 2x = 12 => x = 6
Substitute into Eq. 1: 6 + y = 10 => y = 4
∴ x = 6, y = 4. (Graphically, lines intersect at (6,4)).

CASE BASED QUESTIONS (04 MARKS QUESTIONS)

1. Mr. Manoj Jindal arranged a lunch party. The expenses are partly fixed and partly proportional to the number of guests. The total cost for 7 guests is Rs650 and for 11 guests is Rs 970.



(i) Form a pair of linear equations.
(ii) Find the fixed and variable expenses.
(iii) How much will it cost for 15 guests?
(iv) If Mr. Jindal has a budget of Rs1500, what is the maximum number of guests he can invite?
Solution: (i) x + 7y = 650 (Eq. 1), x + 11y = 970 (Eq. 2)
(ii) Eq. 2 - Eq. 1: 4y = 320 => y = 80. x + 7(80) = 650 => x = 90.
Fixed = Rs90, Variable = Rs80
(iii) Cost for 15= 90 + 15(80) = Rs1290
(iv) Let 'n' be maximum guests. 90 + n(80) = 1500 => 80n = 1410 => n = 17.625. Thus n=17.
2. From Bangalore bus stand, 2 tickets to Malleshwaram and 3 to Yeshwanthpur cost Rs46; 3 to Malleshwaram and 5 to Yeshwanthpur cost Rs74.



(i) Form a pair of linear equations
(ii) Find fare to each place
(iii) Cost of 4 tickets to Malleshwaram and 2 to Yeshwanthpur?
(iv) A tourist has Rs100. How many tickets to Malleshwaram can they buy if they also need one ticket to Yeshwanthpur?
Solution: (i) 2x + 3y = 46 (Eq. 1), 3x + 5y = 74 (Eq. 2)

$\Rightarrow x = 8$. Malleshwaram = Rs8, Yeshwanthpur = Rs10
(iii) $4(8) + 2(10) =$ Rs 52
(iv) Spend Rs10 on Yeshwanthpur. Remaining: Rs 90. Each Malleshwaram ticket is $\frac{90}{8} = 11.25$. So, 11 tickets.

3. A chemist has two solutions: one containing 30% alcohol and another 70%. How much of each must he mix to get 40 L of 40% solution?
(i) Form equations
(ii) Solve for the quantities of each solution.
(iii) If the chemist needs to create 20 L of 50% solution instead, how much of each would he need?

Solution: (i) $x + y = 40$ (Eq. 1), $0.3x + 0.7y = 0.4(40) = 16$ (Eq. 2)
(ii) From Eq 1: $x = 40 - y$. Substitute: $0.3(40 - y) + 0.7y = 16$
$\Rightarrow 12 - 0.3y + 0.7y = 16 \Rightarrow 0.4y = 4 \Rightarrow y = 10$. $x = 40 - 10 = 30$,
30% solution = 30 L and 70% solution = 10 L
For 40 L of 40% solution, the chemist needs 30L of 30% solution and 10L of 70% solution.
(iii) $x + y = 20$, $0.3x + 0.7y = 0.5(20) = 10$. $x = 20 - y$. $0.3(20 - y) + 0.7y = 10 \Rightarrow 6 - 0.3y + 0.7y = 10$
$\Rightarrow 0.4y = 4 \Rightarrow y = 10$. $x = 20 - 10 = 10$, 30% solution = 10 L, 70% solution = 10 L
For 20 L of 50% solution, the chemist needs 10L of 30% solution and 10L of 70% solution.

HIGHER ORDER THINKING SKILLS

1. **Solve:** $\frac{1}{x+y} + \frac{1}{x-y} = \frac{5}{6}$, $\frac{1}{x+y} - \frac{1}{x-y} = \frac{1}{6}$
Solution: $a = \frac{1}{x+y}$, $b = \frac{1}{x-y}$. $a + b = \frac{5}{6}$, $a - b = \frac{1}{6}$, $\Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$. $\frac{1}{2} + b = \frac{5}{6} \Rightarrow b = \frac{1}{3}$.
$x + y = 2$, $x - y = 3$. $2x = 5 \Rightarrow x = \frac{5}{2}$. $y = 2 - \frac{5}{2} = \frac{-1}{2}$.
$\therefore x = \frac{5}{2}, y = \frac{-1}{2}$

2. A sum of Rs 500 is in denominations of Rs 5 and Rs 10 notes. If the total number of notes is 90, find how many of each type.
Solution: Rs 5 notes = x, Rs 10 notes = y. $x + y = 90$, $5x + 10y = 500$. $\Rightarrow x + 2y = 100$. Subtracting: $y = 10$. $x = 90 - 10 = 80$.
$\therefore$ Rs5 notes = 80, Rs10 notes = 10

3. The sum of the ages of father and son is 45. Five years ago, the father's age was 4 times his son's. Find their present ages.
Solution: Father = x, Son = y. $x + y = 45$. $x - 5 = 4(y - 5) \Rightarrow x - 4y = -15$.
Substitute $x = 45 - y$: $45 - y - 4y = -15 \Rightarrow -5y = -60 \Rightarrow y = 12$. $x = 45 - 12 = 33$.
$\therefore$ Father = 33 years, Son = 12years

4. Solve: $\frac{3x}{2y} + \frac{2y}{3x} = \frac{13}{6}$, $\frac{3x}{2y} - \frac{2y}{3x} = \frac{1}{6}$
Solution: $a = \frac{3x}{2y}$, $b = \frac{2y}{3x}$. $a + b = \frac{13}{6}$, $a - b = \frac{1}{6}$. $2a = \frac{14}{6} \Rightarrow a = \frac{7}{6}$. $b = 1$. inconsistent because a & b must be reciprocal. Therefore, no unique solutions.
$\therefore$ The original equations are inconsistent, therefore not solvable.

5. If 6 is subtracted from the numerator and 3 from the denominator of a fraction, it becomes $\frac{1}{2}$. If 3 is added to both, it becomes $\frac{4}{5}$. Find the fraction.
Solution: Fraction $= \frac{x}{y}$. $\frac{x-6}{y-3} = \frac{1}{2} \Rightarrow 2x - 12 = y - 3 \Rightarrow 2x - y = 9$.
$\frac{x+3}{y+3} = \frac{4}{5} \Rightarrow 5x + 15 = 4y + 12 \Rightarrow 5x - 4y = -3.$, $y = 2x - 9$. Substitute: $5x - 4(2x - 9) = -3 \Rightarrow 5x - 8x + 36 = -3 \Rightarrow -3x = -39 \Rightarrow x = 13$. $y = 2(13) - 9 = 17$.
$\therefore \frac{13}{17}$ is the required fraction.